

Tugas Metode Matematika (C)

► Soal Latihan no 2 halaman 56

2). Buktikan : $\Delta \sin(a+bx) = 2 \sin \frac{b}{2} \cos(a + \frac{b}{2} + bx)$

$$\Delta \sin(a+bx) = \sin(a+b(x+1)) - \sin(a+bx)$$

$$\text{Ingat! } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

 $\Rightarrow$ Asal Persamaan :

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\text{misal : } A+B = P \dots (1) \rightarrow A = P - B \dots (3)$$

$$A-B = Q \dots (2) \rightarrow B = A - Q \dots (4)$$

 $\rightarrow$ Dari (2) dan (3)

$$A - B = Q$$

$$P - B - B = Q$$

$$-2B = Q - P$$

$$B = \frac{P-Q}{2}$$

 $\rightarrow$ Dari (1) dan (4)

$$A + B = P$$

$$A + A - Q = P$$

$$2A = P + Q$$

$$A = \frac{P+Q}{2}$$

$$\text{Sehingga, didapatkan : } \sin P - \sin Q = 2 \cos \left[\frac{P+Q}{2} \right] \sin \left[\frac{P-Q}{2} \right]$$

$$\Delta \sin(a+bx) = \sin(a+b(x+1)) - \sin(a+bx)$$

$$= 2 \cos \left[\frac{a+bx+b+a+bx}{2} \right] \sin \left[\frac{a+bx+b-a-bx}{2} \right]$$

$$= 2 \cos \left[\frac{2(a+bx + \frac{1}{2}b)}{2} \right] \sin \left[\frac{b}{2} \right]$$

$$\Delta \sin(a+bx) = 2 \sin \frac{b}{2} \cos(a + \frac{b}{2} + bx) \quad [\text{Terbukti}]$$

► Soal Latihan no 3 halaman 58

3). $2.5.8 + 5.8.11 + 8.11.14 + \dots + 20.23.26$

$$\text{Suku umum : } (2+3x)(5+3x)(8+3x)$$

$$= (10 + 21x + 9x^2)(8+3x)$$

$$= (80 + 30x + 168x + 63x^2 + 72x^2 + 27x^3)$$

$$= (27x^3 + 135x^2 + 198x + 80)$$

$$\Rightarrow A(x)(x-1)(x-2) + B(x)(x-1) + C(x) + D = 27x^3 + 135x^2 + 198x + 80$$

$$\rightarrow x=0 \rightarrow D = 80$$

$$\rightarrow x=1 \rightarrow C + D = 440$$

$$C + 80 = 440$$

$$C = 360$$

$$\Rightarrow x=2 \rightarrow 2B + 2C + D = 1232$$

$$2B + 720 + 80 = 1232$$

$$2B = 1232 - 800$$

$$2B = 432$$

$$B = 216$$

$$\Rightarrow x=-1 \rightarrow -6A + 2B - C + D = -10$$

$$-6A + 432 - 360 + 80 = -10$$

$$-6A = -162$$

$$A = 27$$

$$\Rightarrow 27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80$$

$$\sum_{i=0}^{n=6} 27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80$$

$$\Delta^{-1} 27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80 \Big|_0^7$$

$$\left[\frac{27}{4}x^{(4)} + \frac{216}{3}x^{(3)} + \frac{360}{2}x^{(2)} + 80x^{(1)} \right]_0^7$$

$$= \frac{27}{4} \cdot (7)(6)(5)(4) + \frac{216}{3} (7)(6)(5) + \frac{360}{2} (7)(6) + 80(7)$$

$$= 5670 + 15120 + 7560 + 560$$

$$= 28.910$$

► Soal Latihan no.15 halaman 58

$$15) \cdot \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots \text{ S/d suku ke-} n$$

Suku umum : $\frac{1}{(3x+1)(3x+4)}$

$$\begin{aligned} * \frac{1}{(3x+1)(3x+4)} &= \frac{A}{3x+1} + \frac{B}{3x+4} \\ &= \frac{3Ax + 4A + 3Bx + B}{(3x+1)(3x+4)} \\ &= \frac{(3A+3B)x + 4A+B}{(3x+1)(3x+4)} \end{aligned}$$

$$\Rightarrow 3A + 3B = 0$$

$$A = -B$$

$$\Rightarrow 4A + B = 1$$

$$-4B + B = 1$$

$$-3B = 1$$

$$B = -\frac{1}{3}$$

$$A = \frac{1}{3}$$

$$\begin{aligned} &= \frac{1/3}{3x+1} - \frac{1/3}{3x+4} \\ &= \frac{1}{3} \left[\frac{1}{3x+1} - \frac{1}{3x+4} \right] \\ &= \frac{1}{3} \left[\frac{1}{3} \left(\frac{1}{x+1/3} - \frac{1}{x+4/3} \right) \right] \\ &= \frac{1}{9} \left[\frac{1}{x+1/3} - \frac{1}{x+4/3} \right] \end{aligned}$$

$$\text{Anggap : } f(x) = \frac{1}{x^{(1)} + 1/3} \rightarrow f(x+1) = \frac{1}{x^{(1)} + 4/3}$$

$$\Delta f(x) = f(x+1) - f(x)$$

$$= \frac{1}{x^{(1)} + 4/3} - \frac{1}{x^{(1)} + 1/3}$$

$$\text{Karena } Ux = \frac{1}{9} \left[\frac{1}{x+1/3} - \frac{1}{x+4/3} \right]$$

$$\Rightarrow Ux = -\frac{1}{9} \cdot \Delta f(x)$$

► Deret

$$\begin{aligned} \sum_{x=0}^n Ux &= \sum_{x=0}^n -\frac{1}{9} \Delta f(x) = -\frac{1}{9} \Delta^{-1} \cdot \Delta f(x) \Big|_0^{n+1} \\ &= -\frac{1}{9} f(x) \Big|_0^{n+1} \\ &= -\frac{1}{9} \cdot \left[\frac{1}{x^{(1)} + 1/3} \right]_0^{n+1} \end{aligned}$$

$$\Rightarrow -\frac{1}{9} \left[\frac{1}{n+4/3} - \frac{1}{1/3} \right]$$

$$= \frac{1}{3} - \frac{1}{9n+12}$$

$$= \frac{g(n+1)}{g(3n+4)}$$

$$\text{Jawab} = \frac{n+1}{3n+4}$$

► Soal Latihan no. 9 halaman 58

9) $0 + 10 + 33 + 77 + 150 + \dots$ s/d suku ke- n

U_x	ΔU_x	$\Delta^2 U_x$	$\Delta^3 U_x$
0			
	10		
10		13	
	23		8
33		21	
	44		8
77		29	
	73		
150			

Karena beda tingkat ketiga konstan, maka dengan Teorema Newton diperoleh :

$$U_x = U_0 + x^{(1)} \Delta U_0 + \frac{1}{2!} x^{(2)} \Delta^2 U_0 + \frac{1}{3!} x^{(3)} \Delta^3 U_0$$

$$= 0 + x^{(1)} \cdot 10 + \frac{1}{2} \cdot 13 \cdot x^{(2)} + \frac{1}{6} \cdot 8 \cdot x^{(3)}$$

$$= \frac{4}{3} x^{(3)} + \frac{13}{2} x^{(2)} + 10 x^{(1)}$$

$$\sum_{x=0}^{n-1} U_x = \Delta^{-1} \left\{ \frac{4}{3} x^{(3)} + \frac{13}{2} x^{(2)} + 10 x^{(1)} \right\} \Big|_0^n$$

$$= \frac{1}{3} x^{(4)} + \frac{13}{6} x^{(3)} + 5 x^{(2)} \Big|_0^n$$

$$= \frac{1}{3} (n)(n-1)(n-2)(n-3) + \frac{13}{6} (n)(n-1)(n-2) + 5 (n)(n-1) - \left[\frac{1}{3} (0) + \frac{13}{6} (0) + 5 (0) \right]$$

$$= \frac{n(n-1) \left[2(n-2)(n-3) + 13(n-2) + 30 \right]}{6}$$

$$= \frac{n(n-1) \left[2n^2 - 10n + 12 + 13n - 26 + 30 \right]}{6}$$

$$= \frac{n(n-1) (2n^2 + 3n + 16)}{6}$$

► Soal Latihan no 2b halaman 58

2). b). $1^4 + 2^4 + 3^4 + \dots + n^4$

$$\Rightarrow A(x)(x-1)(x-2)(x-3) + B(x)(x-1)(x-2) + C(x)(x-1) + D(x) + E = x^4$$

$$\rightarrow x=0 \rightarrow E=0$$

$$\rightarrow x=3 \rightarrow 6B + 6C + 3D = 81$$

$$\rightarrow x=1 \rightarrow D + 0 = 1$$

$$6B + 42 + 3 = 81$$

$$D = 1$$

$$6B = 36$$

$$B = 6$$

$$\rightarrow x=2 \rightarrow 2C + 2D = 16$$

$$2C = 14$$

$$\rightarrow x=-1 \rightarrow 24A - 6B + 2C - D = 1$$

$$C = 7$$

$$24A - 23 = 1$$

$$24A = 24$$

$$A = 1$$

Suku Umum : $x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)}$

$$\sum_{i=0}^n x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)} = \Delta^{-1} x^{(4)} + 6x^{(3)} + 7x^{(2)} + x^{(1)} \Big|_0^{n+1}$$

$$= \frac{1}{5} x^{(5)} + \frac{3}{2} x^{(4)} + \frac{7}{3} x^{(3)} + \frac{1}{2} x^{(2)} \Big|_0^{n+1}$$

$$= \frac{6(n+1)(n)(n-1)(n-2)(n-3) + 45(n+1)(n)(n-1)(n-2) + 70(n+1)(n)(n-1) + 15(n+1)n}{30}$$

$$= \frac{n(n+1) \left[6(n^3 - 6n^2 + 11n - 6) + 45(n^2 - 3n + 2) + 70n - 70 + 15 \right]}{30}$$

$$= \frac{n(n+1) \left[6n^3 + 9n^2 + n - 1 \right]}{30}$$

$$= \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30}$$